JOINT NUMERICAL WEATHER PREDICTION UNIT

OFFICE NOTE #7

On the Problem of Comparing Station Pressures at Varying Elevations

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1. Introductory Remarks

The problem discussed in this paper is among the oldest in modern meteorology. Its solution has been hampered by consistently vague statements of the problem. V. E. Jakl, in an intra-bureau communication in 1952, has indicated the problem to be a comparison of station pressures which would conveniently yield horizontal pressure gradients. He said, however, "It was realized from the very beginning of meteorological science that pressures had to be reduced to some fixed level in order to obtain pressure gradients, even though the deficiencies of such a system were recognized from the beginning." This is an expression of a crucial point of departure from the real problem. This departure has been universally shared by early writers on the subject.

The present system of barometric reductions in use in the United States is based directly on a monumental work by Bigelow (1902). Bigelow's starting point was a conception expressed earlier by Ferrel (1885,1886), who recognized the many difficulties involved in a system of reductions of the barometer to sea level. Ferrel (1886, p.221) stated the optimum solution of the problem as he saw it, "The best that can be done is to imagine that the space between the station and sea level is occupied by air, and that this has the same temperature as the air would have if the mountain or plateau were away." Bigelow implicitly adopted this view. Our present system of reductions, based on his work, are combinations of station pressures and temperatures designed to define the hypothetical air columns below the stations.

It is not the author's intent to discuss the philosophical absurdity of Ferrel's statement of the optimum solution of the problem. There is a discussion of the philosophical difficulties in his own work (1886). While Chief of the U. S. Weather Bureau, Prof. C. F. Marvin wrote, in part, to the Toronto symposium on barometric reductions held in 1921 (Meisinger, 1921), "It is well known that, strictly speaking, a reduction to sea level of continental observations is wholly a visionary and hypothetical thing, since to effect the reduction we must necessarily assume that an atmosphere exists below the station whose observations are to be reduced. We assign to this atmosphere a given temperature and moisture content, density, etc., and our reduction depends upon these assumptions. Since the intervening atmosphere has no existence our assumptions are necessarily fictitious, and the final result of the so-called reduction to sea level has in reality no meaning". Nor is it the author's intent to discuss the insuperable difficulties involved in the determination of pressures reduced to sea level according to Ferrel's statement, when both dynamic and static considerations are taken into account.

2. A Precise Statement of the Problem in Terms of its Optimum Solution

Before going further in the discussion, the problem must be stated more precisely. What sort of comparison of station pressures is wanted in meteorology? The field of station pressures can give three basic types of information..

1. Comparisons of the weight of the atmosphere above the various stations. Such a comparison has little interest in itself. A "reduction" of the field of station pressures to a level plane, such as the system currently in use in the United States, in indicated for this type of comparison.
2. Purely statistical parameters with which to forecast future states of the atmosphere. The present system of reductions is again indicated here, for meteorology has some fifty years of accumulated statistics, or experience, with it. The indication is superficial, however. Variables which have clearer physical meanings than the presently used pressure reduced to sea level could be handled more intelligently in statistical studies, and in the forecasters' experience. When such variables are found, the course of the weather services should be to collect statistics and experience with it, rather than to compound the handicaps of the past by continuing to use the method of reduction now in use.
3. Measures of the horizontal pressure gradient force field. This is a universally useful type of information. Because of the partially balanced nature of atmospheric flow, and because, as we shall see, the answer with this as the precise objective does

not differ radically from the answer currently in use, such measures will give in part the type of information enumerated under 1. and 2. above.

We will adopt as the specific problem at hand, to find a means of comparison of pressures measured at the various stations which will conveniently give us a measure of the horizontal pressure gradient force.

3. Representations of the Pressure Gradient Force Field

Weather reporting stations are placed at irregular finite intervals which are small compared with the horizontal dimensions of the large-scale atmospheric systems, but which are large compared to the dimensions of detailed topographical features, particularly in the mountainous regions of the United States. There is no need, however to consider topography in all its detail. A smooth surface may be passed through all the stations in the United States, and we may then confine our attention to this imaginary surface, rather than the actual topographical surface. For convenience, we will distinguish the topography of the smooth surface passing through all the weather stations from the actual topography by calling it the "meteorological topography." We will use the symbol H_b, which is commonly used for station elevation, as the height of the meteorological topographic surface above sea level. H_b, then will be a continuous variable in x and y. A list of station elevations may be considered a sampling of H_b at small finite intervals of x and y. The disregard for the detail of the actual topography of the ground is necessitated by the lack of coinciding samples of surface pressure and height above sea level at intervals small enough to handle such detail.

For mathematical convenience, a new variable, h, will be defined. The variable h is the height above the meteorological topographic surface, i.e.,

$$h(x,y,z) = z - H_b(x,y) \tag{1}$$

The horizontal pressure gradient force is

$$\alpha \nabla_z p$$

where the subscript $(\ )_z$ denotes differentiations at constant height, z, above sea level, α is specific volume, and p is pressure. A direct measure of horizontal pressure force would involve measurements of horizontal space derivatives holding z constant. This is obviously impossible to do on an operational basis, except over level surfaces. The problem is to obtain a measure of the horizontal pressure gradient force by comparing quantities measured at constant h, in particular by comparing quantities measured on the surface h = 0, which is the meteorological topographic surface. In principle the problem may be solved by a transformation of coordinates: namely, transformation of the derivative, ∇_z, in x, y, z-space into derivatives in x, y, h-space. The transformation formula with which to accomplish this is

$$\nabla_z = \nabla_h - \nabla_h z \frac{\partial}{\partial z} \tag{2}$$

Applying the transformation formula (2) and making use of the hydrostatic approximation, we find that the horizontal pressure gradient force is

$$\alpha \nabla_z p = \alpha \nabla_h p + \nabla_h gz \tag{3}$$

if the variation of gravity, g, is neglected.

Equation (3) is a solution to the problem as we have stated it, although not a practical one. It is easy to show that in the atmosphere the two terms on the right-hand side of equation (3) are each in general much larger than their sum, so that neither can be neglected. In order to find the horizontal pressure gradient force by means of equation (3) it would be necessary to find the small sum of two large vectors. This would be an intolerable procedure in forecasting practice.

At this point it is in order to inquire into the practical aspects of the problem. What would be considered a tolerable representation of the horizontal pressure gradient force field? None comes to mind other than by means of a scalar function whose gradient is closely related to the pressure gradient force. This is the form in use today, the scalar function being pressure reduced to sea level. Since the second term in equation (3) is not an exact differential, the pressure gradient force cannot be set equal to the gradient of a scalar function. One could, however, find two scalar functions from which the pressure gradient force could be measured. For example, one could set

$$\alpha\nabla_h p + \nabla_h gz = f\nabla_h F \qquad (4)$$

The author has not considered seriously the boundary conditions for solving equation (4) for f and F, nor methods of solving it. The integration could not be made into a boundary-value problem. Thus, if the region of integration were restricted to continental areas, with the shore line as boundary, one could not set $f = \alpha$ and $F = p$ all around the shore line. The integration would thus have to extend over the entire region of meteorological interest, and over oceanic areas f and F would not carry the physical significance which α and p do.

Perhaps a more interesting pair of scalar functions would be a stream function and velocity potential. Thus,

$$\alpha\nabla_h p + \nabla_h gz = \nabla_h S_2 + \nabla_h S_1 \times k \qquad (5)$$

where k is a unit vertical vector. If one set the boundary entirely on oceanic surfaces, and set S_2 equal to the height of an isobaric surface near sea level, S_2 over oceanic surfaces would very nearly coincide with the height of the isobaric surface, and thus would have equal physical significance. Perhaps of more interest, S_2 would entirely (i.e., without reference to S_1) define the vorticity of the geostrophic wind except for slight variations in the Coriolis parameter. One could then observe the effects of a massif on cyclonic and anticyclonic systems as they crossed the massif. The field of S_1 would be an adjunct to the field of S_2, and would yield at a glance, or by precise measure, that part of the horizontal pressure gradient force not contained in the gradient of S_2.

The determination of the fields of S_1 and S_2 would be relatively straightforward. Taking the divergence of equation (5),

$$\nabla_h\alpha \cdot \nabla_h p + \alpha\nabla_h^2 p + \nabla_h^2 gz = \nabla_h^2 S_2 \qquad (6)$$

we have a Poisson equation in S_2. Taking the curl of equation (5)

$$(\nabla_h p \times \nabla_h \alpha) \cdot k = \nabla_h^2 S_1 \qquad (7)$$

we have a Poisson equation in S_1. As before, the boundary values of S_2 would be the geodynamic height of an isobaric surface near sea level. The boundary conditions for the solution of equation (7) should then be that the derivative of S_1 normal to the boundary be zero.

4. A point-wise computation of a single scalar function

In the past, the solutions outlined in the preceding section have not been feasible, due to the immensity of the computational problem. With the present availability of high-speed computing machinery the computational problem is no longer a serious obstacle, at least in the United States. The calculations are trivial when compared to the calculations presently being done in the numerical prediction of weather. There would be some problem in determining representative temperatures at the ground. For example, diurnal radiation effects on the variation of $\tilde{\alpha}$ should not be included. These problems, however, are surely not insurmountable. A more serious immediate problem would be communications. The computing point would have to collect data, process it, and then disseminate it on a world-wide basis. In some areas of the world, where adequate computational facilities are not anticipated in the near future, and which are relatively inaccessible to weather communications, the only adequate solution to the problem will remain one in which only variables measured at the station go into the determination of the "reduction to sea level".

Even with the problem so restricted, a more enlightened approach should yield an improvement over the solution given by the classical approach. The problem then reduces to finding a scalar function, determined station-wise, whose gradient closely approximates the horizontal pressure gradient force. Without tracing the line of reasoning which led to the following form, we may state that

$$\alpha\nabla_z p = \nabla_h[gz - g\bar{z} + \alpha'''(p-P)] + \alpha'\nabla_h P + \alpha''\nabla_h(p-P) - (p-P)\nabla_h\alpha''' \qquad (8)$$

where g is gravity

- z is elevation
- $\bar{z} = \bar{z}(p)$ is a mean height-pressure relationship such as the U. S. Standard Atmosphere
- p is pressure
- α is specific volume
- $\bar{\alpha}$ is the specific volume in the mean atmosphere defined by $\bar{z}(p)$.
- $\alpha' = \alpha - \bar{\alpha}$ is the variation of specific volume about the mean.

α'' is the variation of specific volume about its daily mean.

α''' is the variation of the daily mean specific volume about its mean as defined by $\bar{z}(p)$.

P is a function of x and y whose choice will be discussed below. It is to be noted that according to our definitions,

$$\alpha = \bar{\alpha} + \alpha'$$

$$\alpha' = \alpha'' + \alpha'''$$

From the arrangement of equation (8), our intention should be clear. Our pressure "reduced to sea level" will be

$$\bar{\alpha}_o^{-1}[gz - g\bar{z} + \alpha'''(p - P)] \qquad (9)$$

The $\bar{\alpha}_o$ appears in order to give the "reduction" the dimensions and general magnitude of a sea level pressure. The $\bar{\alpha}_o$ is some representative value of specific volume at sea level, perhaps taken from the mean atmosphere defined by $\bar{z}(p)$.

The variable $P(x,y)$ should then be chosen so as to minimize in some sense the vector field,

$$\alpha'\nabla_h P + \alpha''\nabla_h(p-P) - (p-P)\nabla_h\alpha''' \qquad (10)$$

As long as the gradients of the chosen function $P(x,y)$ are not generally larger than the gradients of p, the size of the middle term in the error expression (10) is fixed insofar as it does not otherwise depend importantly upon the choice of the function $P(x,y)$. It only remains, then, to determine the function P in such a way that its gradients are sufficiently small to keep the first term in the expression (10)within bounds; and yet in such a way that its gradients are sufficiently large to integrate everywhere into values of P sufficiently close to station pressure to keep the last term in the error expression (10) within bounds. Figure 1 represents a crude attempt to accomplish this. Figure 1 displays the heights in the U. S. Standard Atmosphere corresponding to pressures equal to P.

Figure 2 is the new pressure "reduced to sea level" (π) over the American Rockies and parts of the Great Plains. Figure 3 is the reported pressure reduced to sea level according to the classical system presently used. Note that the new method does not produce the two small-scale highs over western Montana and northwestern Colorado. It is also to be noted that with the new method the low center in the northern Great Plains is in closer agreement with the reported wind field. In these respects the new method is undoubtedly an improvement. The old method, however, seems to produce better agreement with the winds over Iowa, Missouri, Kansas, and Nebraska. Whether this represents a real improvement in the definition of the horizontal pressure gradient force is at least open to question. There is the further question as to whether it would not be wiser to accept a generally high more homogeneous quality over the entire chart rather than a very high quality over some areas and very poor quality over others.

In any case, improvements over the quality in Figure 2 could undoubtedly be obtained by a more careful construction of P(x,y).

In order to verify the importance of the last term in the expression (9) for the "reduction to sea level", the altimeter setting for the same case was plotted and analyzed. The altimeter setting is very nearly equal to $g\bar{\alpha}_o^{-1}(z-\bar{z})$. The result, in Figure 4, shows a very rough field which reflects too closely the topographic gradients. Because of the effort involved, only one calculation of the new reduction was made, which is shown in Figure 2. The case was an unusually warm July day. Since no great effort was required, we set out to determine whether the last term in expression (9) was equally important on a cold day. Figures 5 and 6 show that on a cold day the altimeter setting may actually be an improvement over the present system.

5. Concluding Remarks

Bigelow (1902) realized that in the years following the adoption of his system much more data would be amassed. He suggested that after 20 years the problem be re-examined in the light of data which would then be available. As a result of a suggestion made at the Chicago meeting of the American Meteorological Society in 1920, a symposium on the reduction of barometric pressures in the United States and Canada was held in 1921. The symposium (Meisinger, 1921) was attended by W. J. Humphreys, C. F. Brooks, C. L. Meisinger, J. Patterson, and H. J. Cox. Communications from C. F. Marvin, A. McAdie, and W. G. Reed were read before the group. A resolution was drawn up and adopted by the symposium which read, in part, "...be it resolved by the American Meteorological Society, assembled at Toronto, Canada, this 29th day of December 1921:

"1. That the time is now opportune for a re-examination of the barometric methods employed in the United States and Canada with a view to the possible improvements of pressure reductions.

"2. That such investigations should include an examination of --

...

"(d) The nature and cause of the barometric discrepancies noted above and their true values.

..."

The importance attached to the problem in 1921 may be surmised from the emminence of the men in attendance. The system of comparison of station pressures then in use was not considered the most satisfactory attainable. Thirty-six years later American barometry is still using that system.

This paper clearly indicates new lines of attack, and offers evidence that these new lines of attack will be fruitful. In particular, the proposed lines of attack brings out "...the nature and cause of the barometric discrepancies...". It is hoped that this paper will stimulate further work in the field of American barometry, to bring to a final conclusion the long search for the optimum solution to the problem of comparing barometer readings taken at varying elevations.

ACKNOWLEDGEMENTS

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their time and special skills they lent to the study in analyzing a set of maps which were background material for this investigation.

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LEGENDS

Figure 1. Heights in the U. S. Standard Atmosphere corresponding to the function $P(x,y)$.

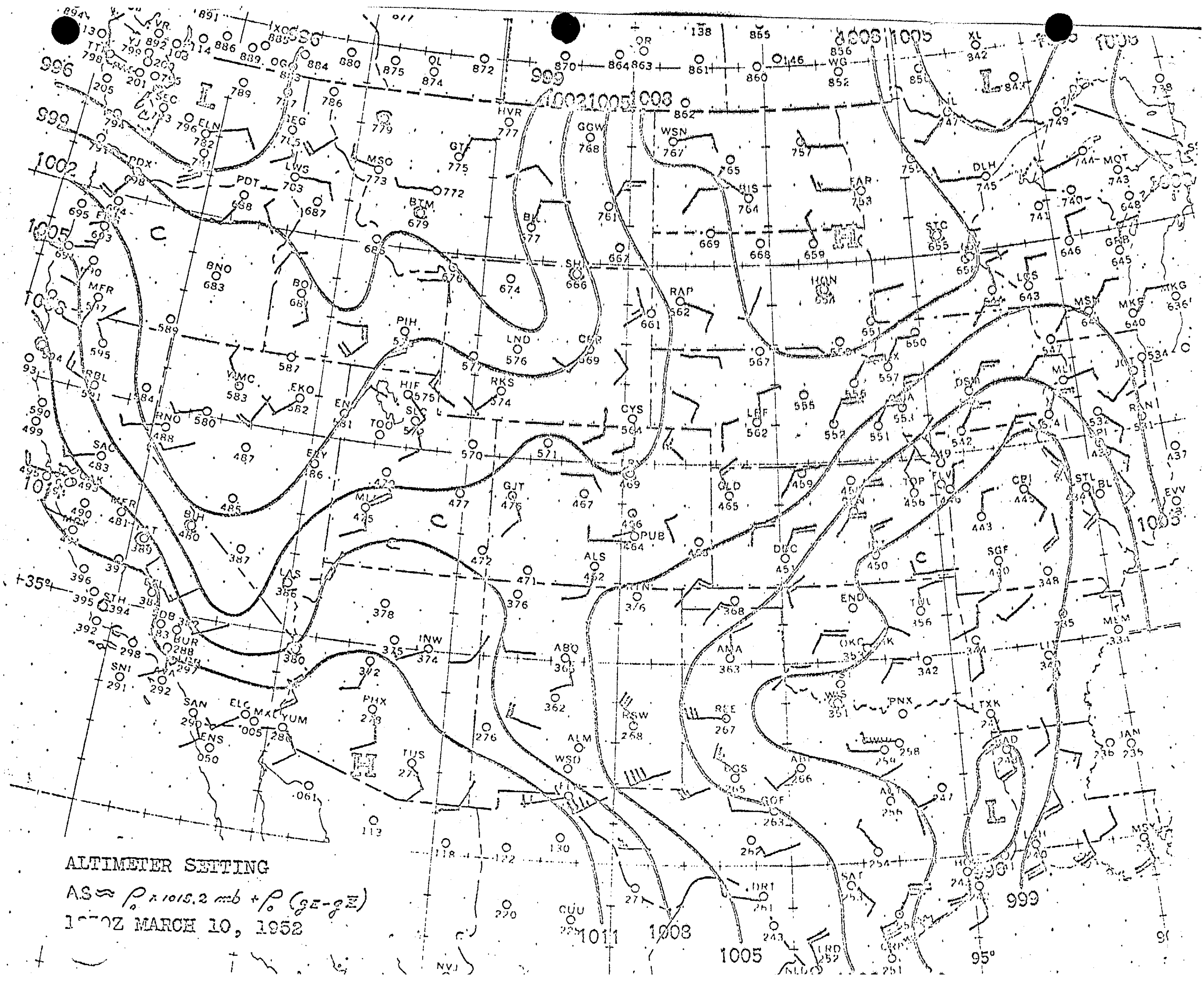
Figure 2. A proposed pressure "reduced to sea level", for 0630GCT July 22, 1952. α''' was defined by means of surface temperatures observed 12 hours apart.

Figure 3. Station pressure classically reduced to sea level for 0630GCT July 22, 1952.

Figure 4. Altimeter settings for 0630GCT July 22, 1952.

Figure 5. Station pressure classically reduced to sea level for 1230GCT March 10, 1952.

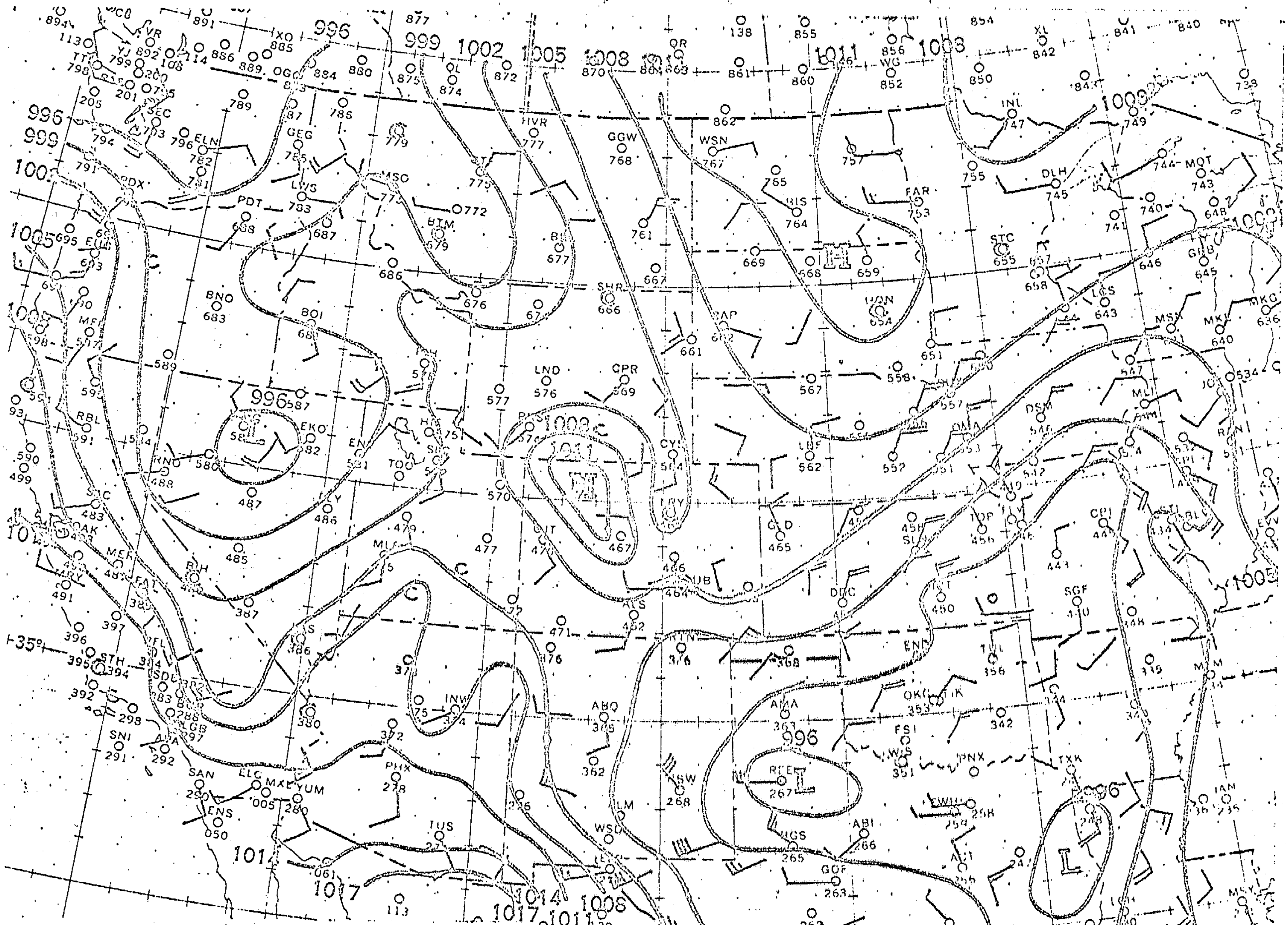
Figure 6. Altimeter settings for 1230GCT March 10, 1952.



ALTIMETER SETTING

$AS \approx \rho_o \times 1015.2\ mb + \rho_o (gz - gZ)$

1[illegible]0Z MARCH 10, 1952



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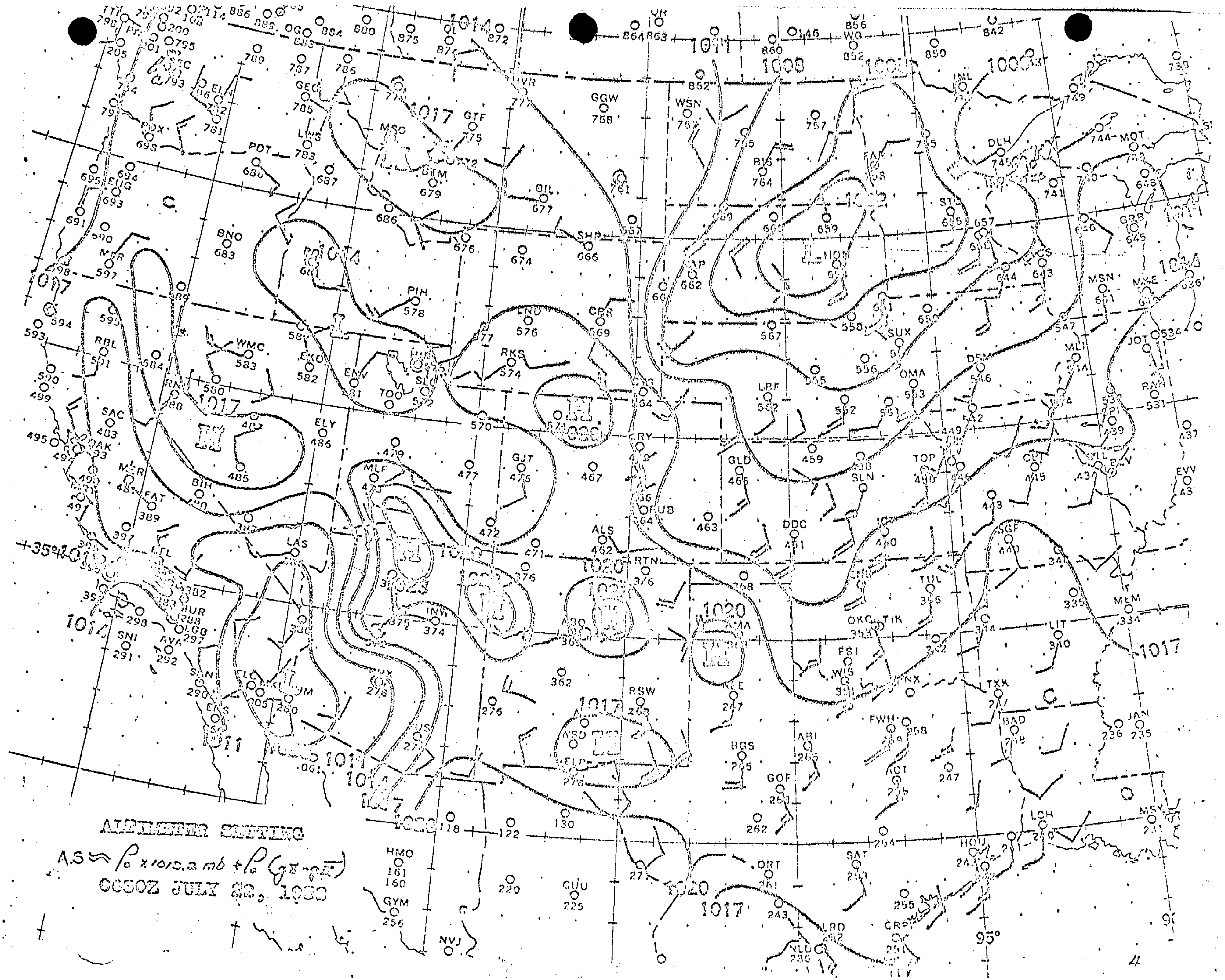
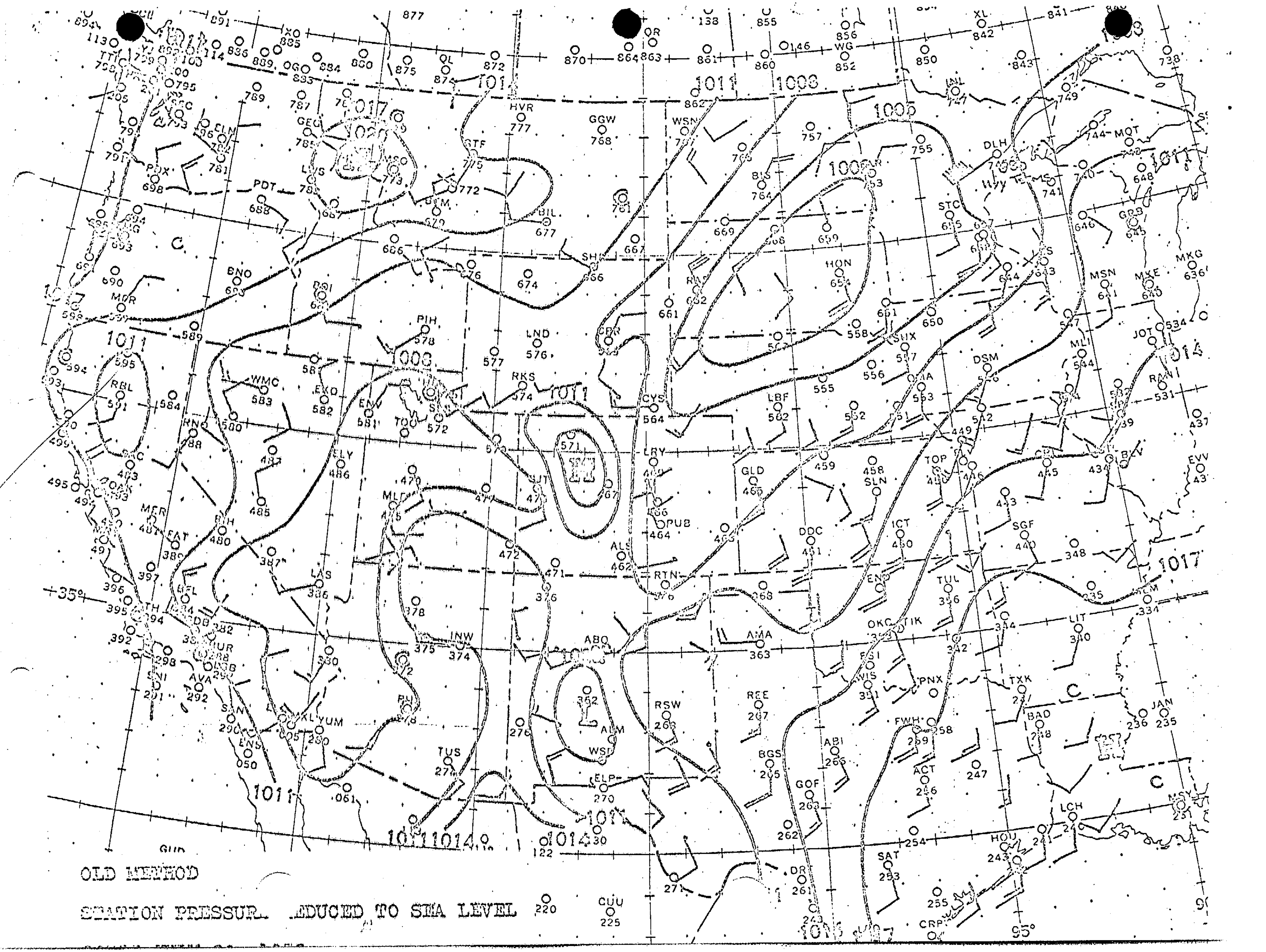
STATION PRESSURE REDUCED TO SEA LEVEL



ALTIMETER SETTING

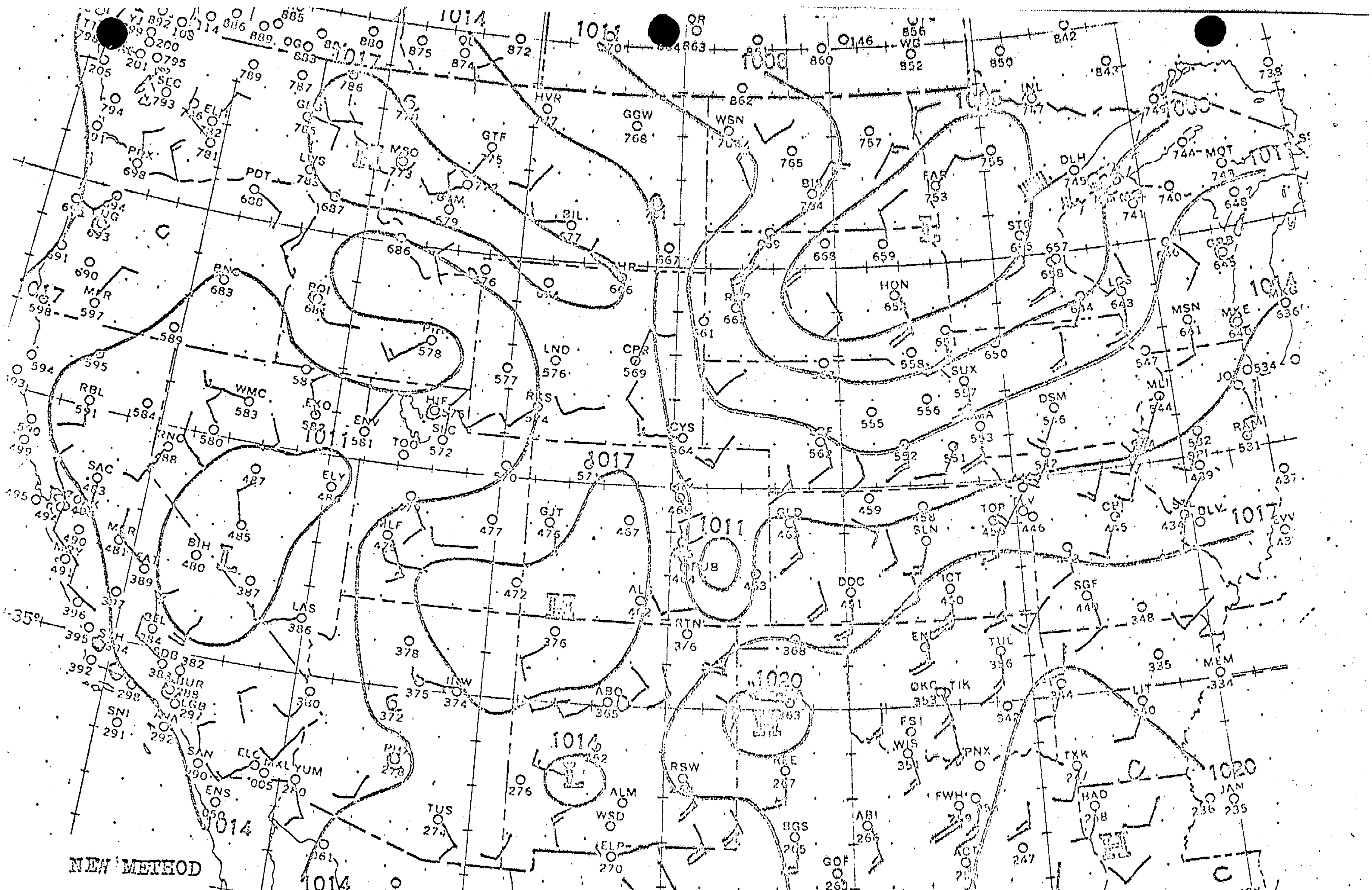
$A.S \approx \rho_0 \times 1013.2\ mb + \rho_0 (g\bar{z} - g\bar{Z})$

0030Z JULY 22, 1980



OLD METHOD

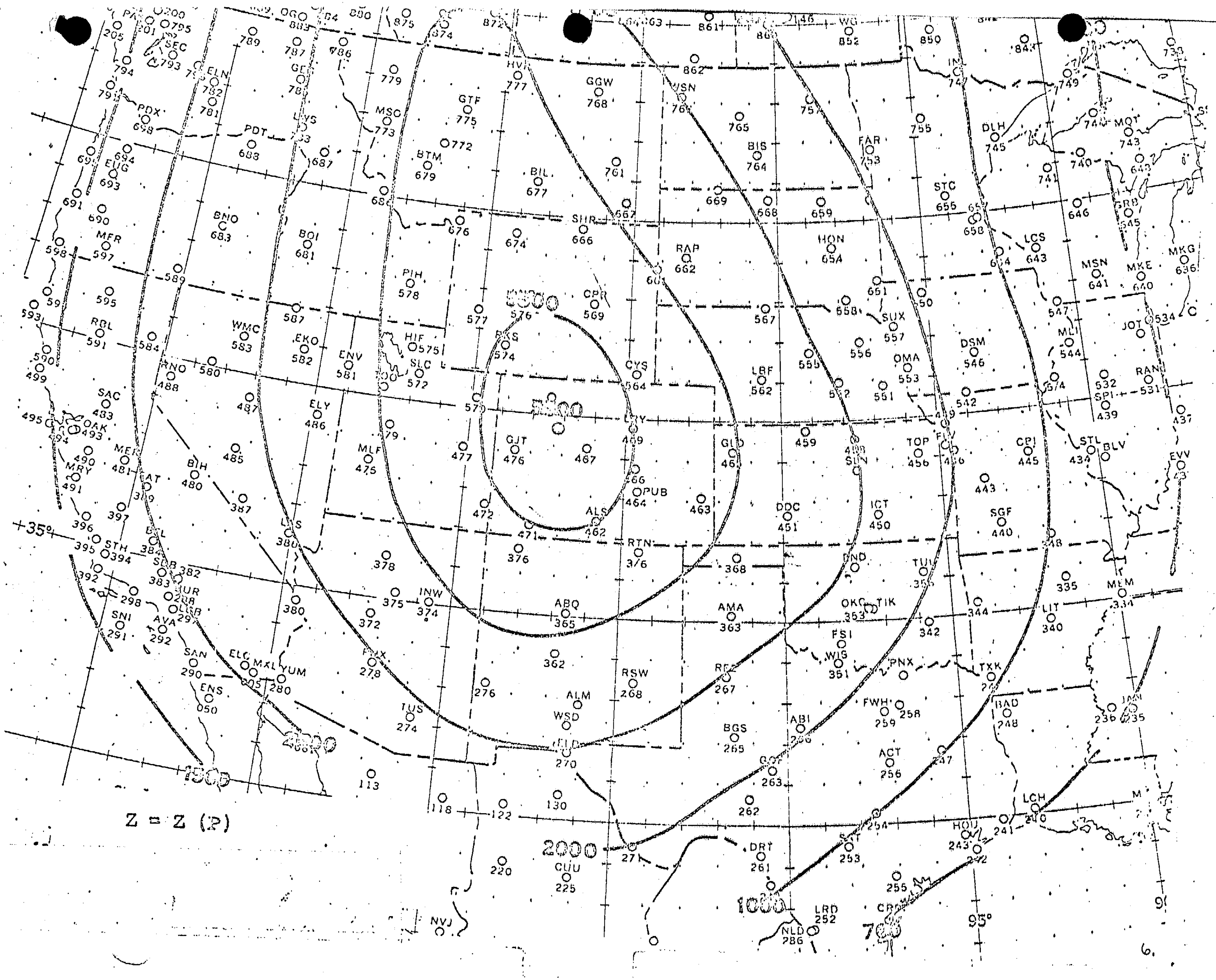
STATION PRESSUR. .EDUCED TO SEA LEVEL



COMPARISON OF STATION PRESSURES BY MEANS OF A REPRESENTATION OF THE HORIZONTAL PRESSURE GRADIENT FORCE AT THE GROUND

$\Pi \approx \rho_0 \times 1013.2\ mb + \rho_0 \left[gz - g\bar{z} + \alpha'' (p - P) \right]$

0630Z JU[illegible] 22, 1952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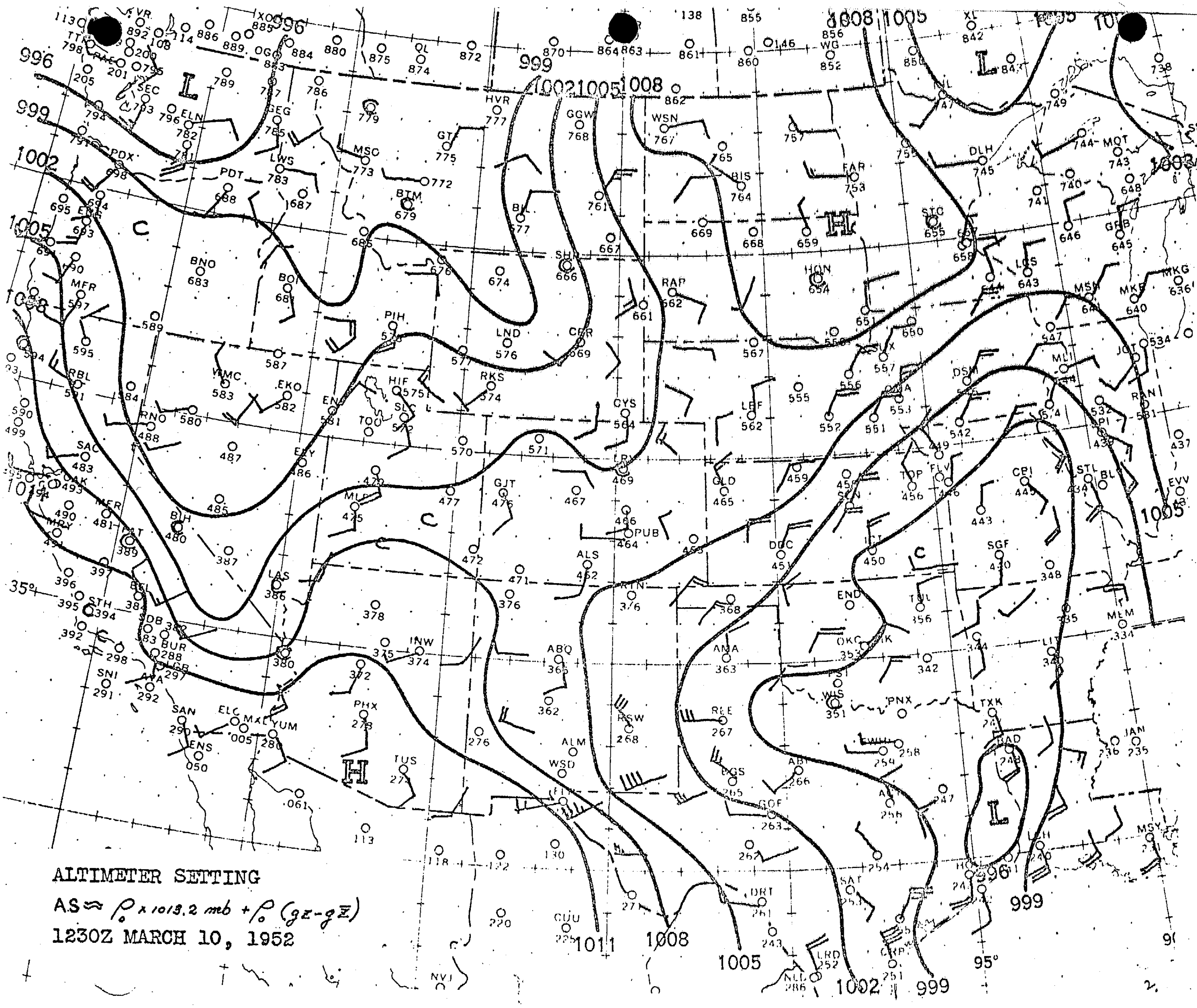
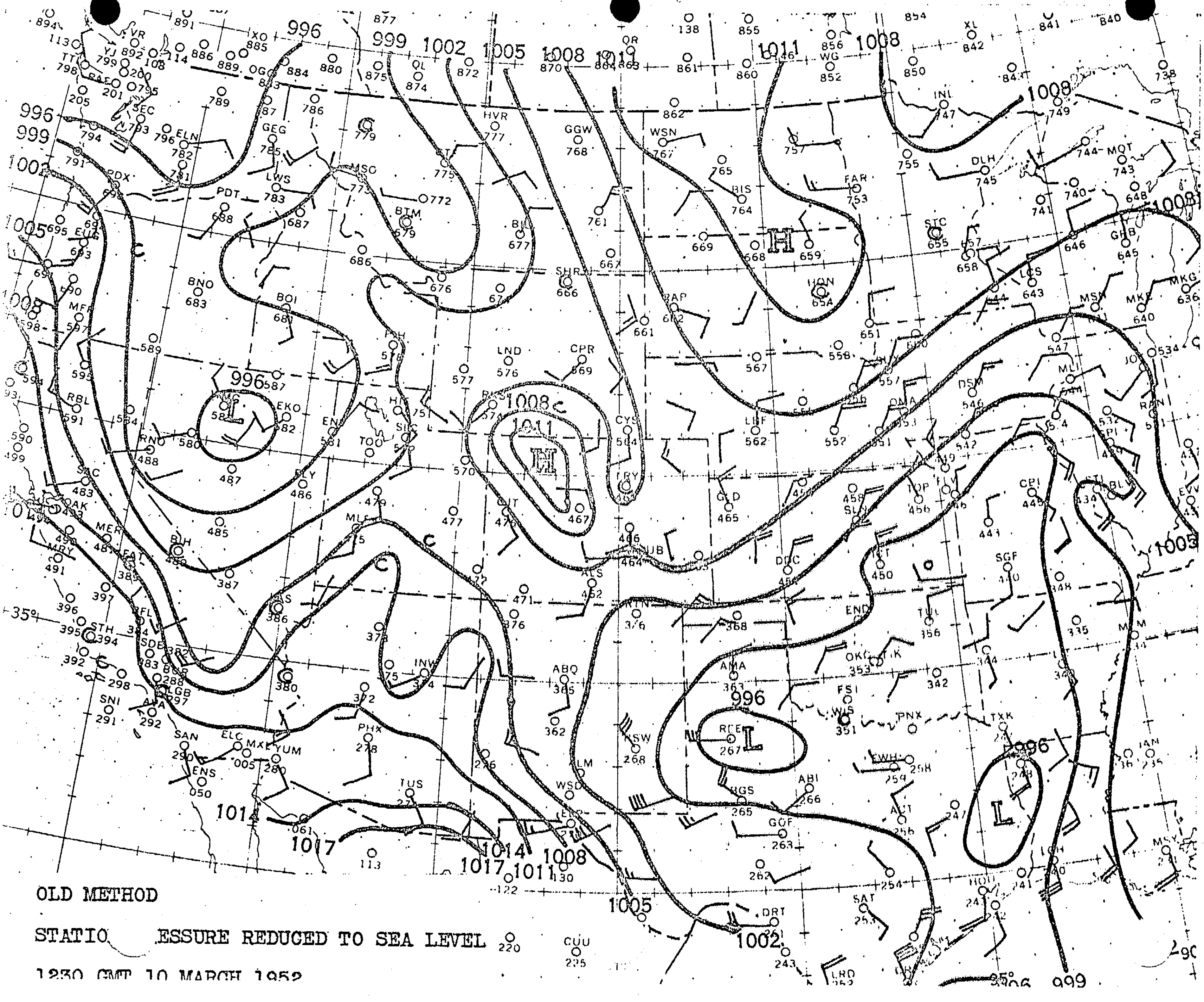
Z = Z (P)



ALTIMETER SETTING

$AS \approx \rho_0 \times 1013.2\ mb + \rho_0 (gz - g\bar{Z})$

1230Z MARCH 10, 1952



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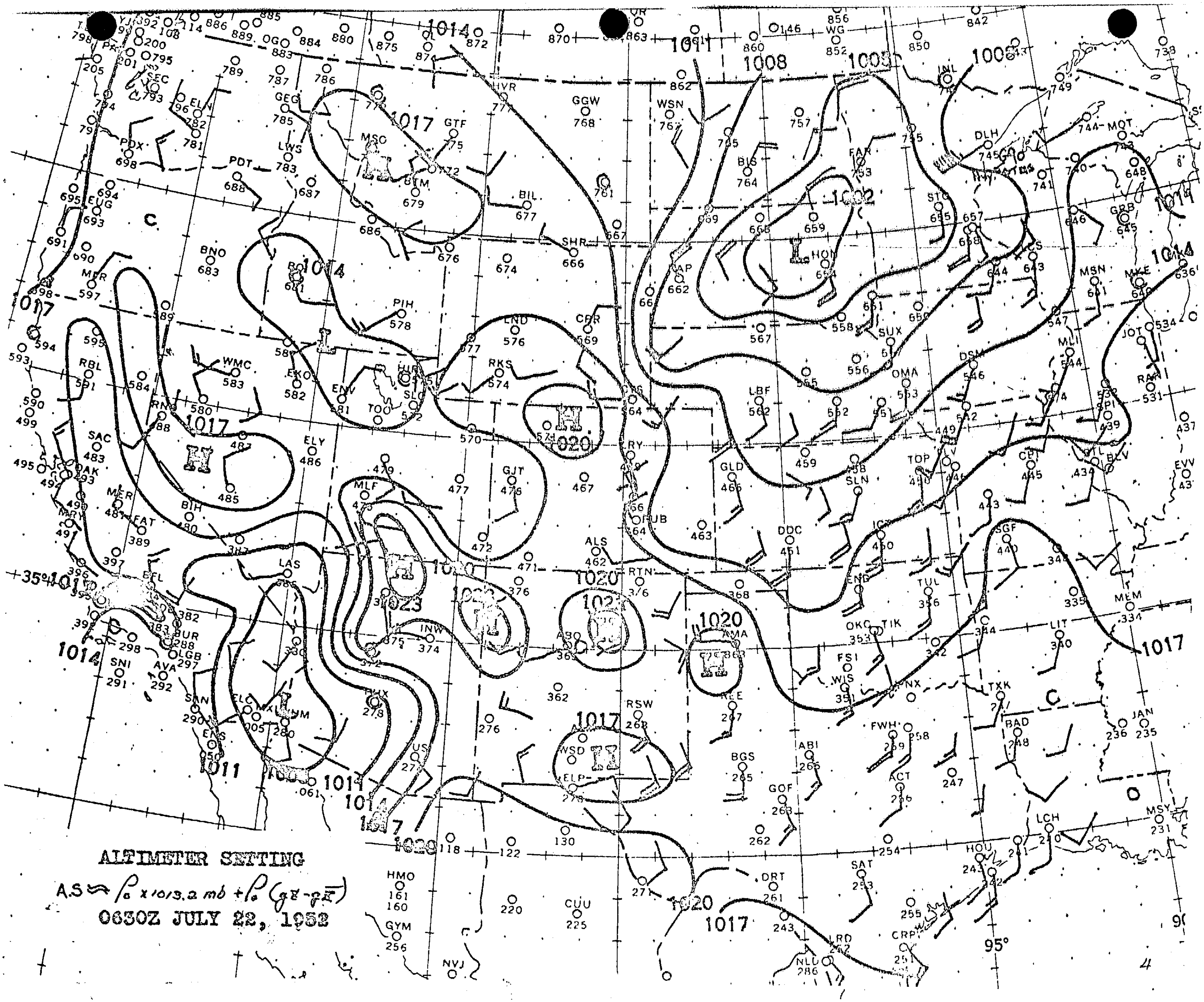
STATIO ESSURE REDUCED TO SEA LEVEL

1230 GMT 10 MARCH 1952

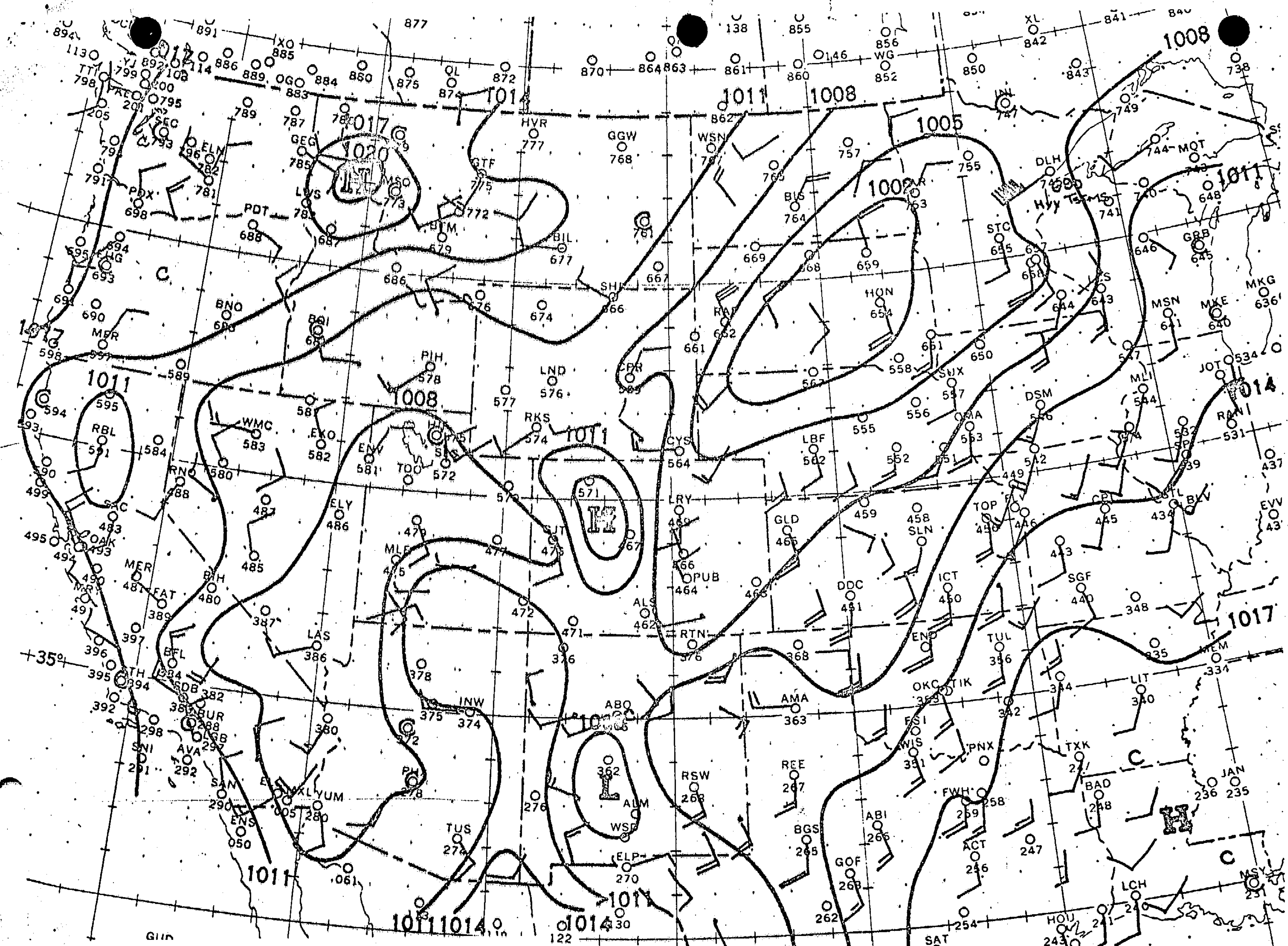
FIG.



ALTIMETER SETTING

$A.S \approx \rho_0 \times 1013.2\ mb + \rho_0 (g\bar{z} - g\bar{z})$

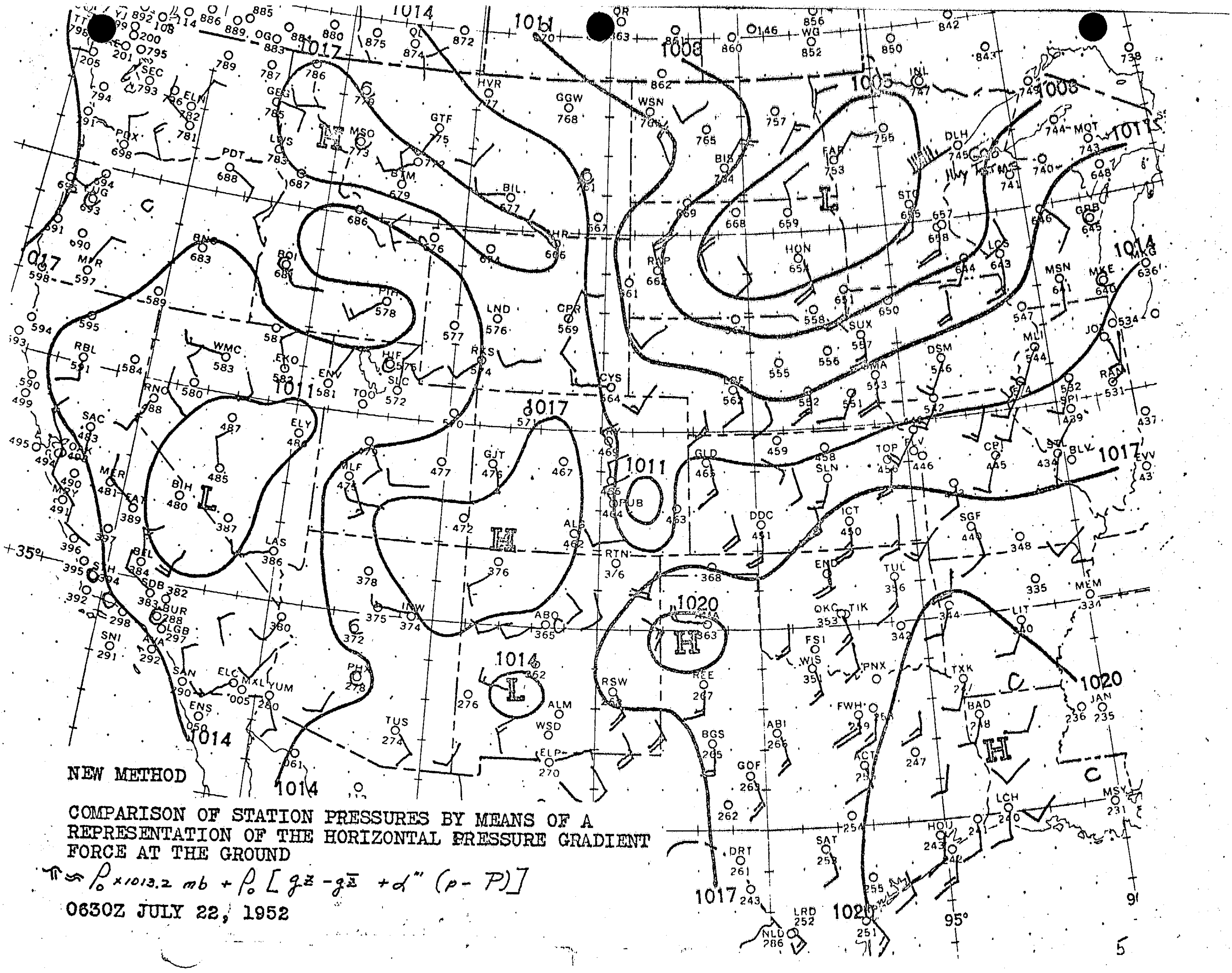
0630Z JULY 22, 1952



OLD METHOD

STATION PRESSU[R]E [R]EDUCED TO SEA LEVEL

0630Z JULY 22, 1952

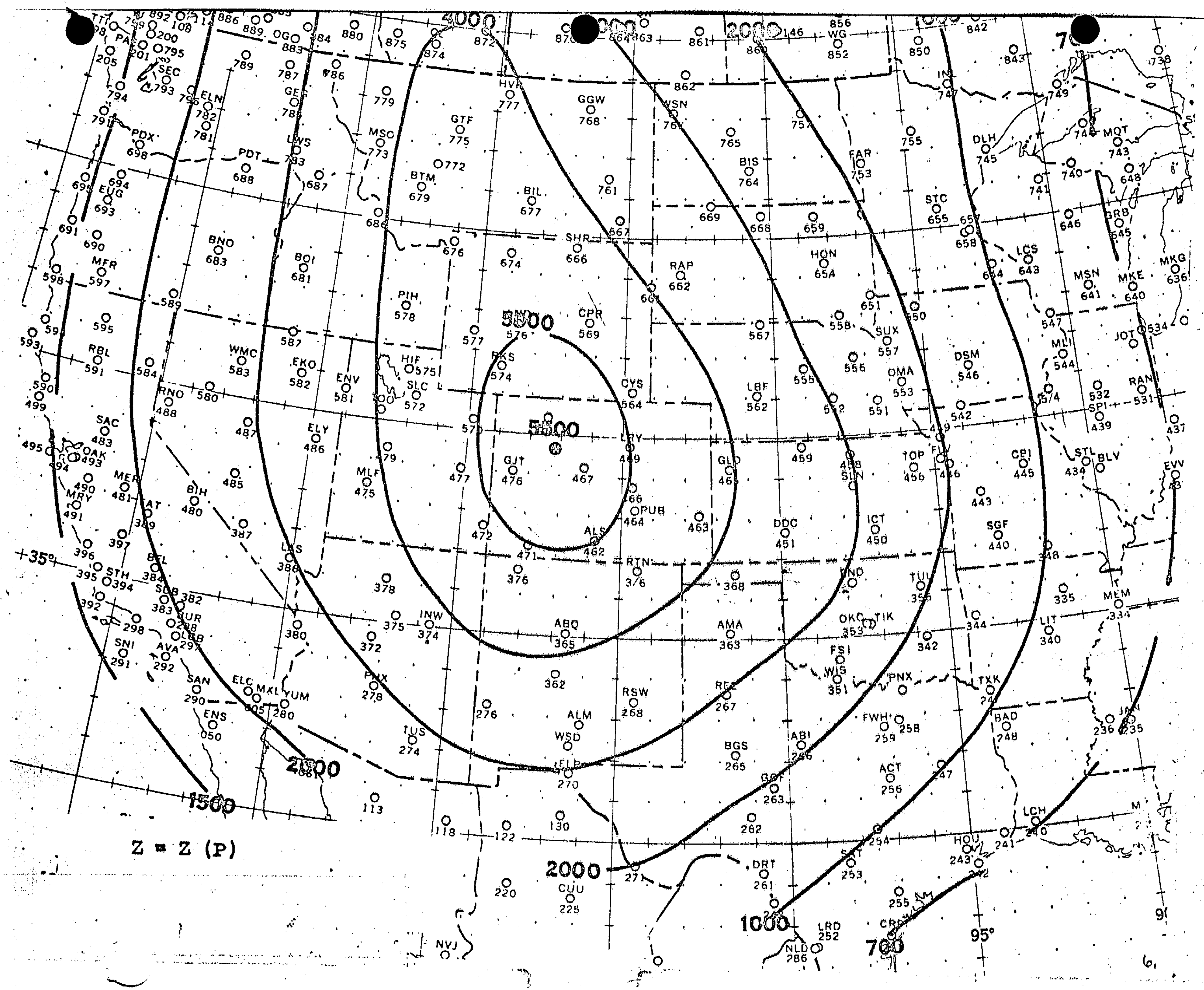


NEW METHOD

COMPARISON OF STATION PRESSURES BY MEANS OF A REPRESENTATION OF THE HORIZONTAL PRESSURE GRADIENT FORCE AT THE GROUND

$$\Pi \approx \rho_0 \times 1013.2\ mb + \rho_0 \left[g z - g \bar{z} + \alpha'' (p - P) \right]$$

0630Z JULY 22, 1952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